Tourism sector recovery plan for airlines

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Overview

Introduction

- One of the key aspects of tourism is movement of people from one place to another.
- Various reasons for travel: business & leisure
- Transport is used to travel
- Three components for tourism transport:
  1. Travel to a destination
  2. Travel at the destination
  3. Travel from the destination (back to place of residence)

Air transport

Comes in different size or carrying capacity

Region	Jul-Sept 2020	Jul-Sept 2021	Difference	% change
International	582	289 743	289 161	49 684%
Regional	0	37 965	37 965	
Domestic	630 845	1 445 885	815 040	129%
Unscheduled	22 666	10 668	-11 998	-53%
Total	654 093	1 784 261	1 130 168	173%

Tourism Sector Recovery Plan COVID-19 Response

- “Ignite” the tourism sector.
- Optimise profit through reduction of taxes on available seats (suggested).

Questions:

1. What changes need to be implemented?
2. How many seats need to be sold and the cost per seat, taking note that due to COVID-19 regulations, airlines are not permitted to carry at full capacity?
3. How can airlines find ways to increase their profit using the available seats under the given COVID-19 regulations and taking into account any invariants?

- Question 1 is too general.
- Question 2 seems more manageable.
- Question 3 is potentially tied to Question 2.

Study Group interpretation of the Representative's questions:

- How can we optimise the number of seats sold?
- How can we optimise the cost per seat?

Solution Approach

We want to:

- maximise a revenue;
- analyse ticket distribution such that the maximum revenue is achieved;
- consider alternative ways of optimising revenue.

Optimal Control Approach

Simplified Model

We assumed:

- a plane with seven (7) seats;
- three (3) different ticket classes.

Let

- x_j = number of seats/ tickets of class j that should be allocated
- $R_j(x_j)$ = revenue generated by sales of x tickets of class j

where $j = 0, 1, 2$.

# tickets x_j	$R_0(x)$	$R_1(x)$	$R_2(x)$
0	0	0	0
1	1200	1500	2000
2	2400	3000	4000
3	3600	4500	4900
4	5000	5600	6000
5	8000	7700	7000
6	8500	8700	9000
7	9200	10000	11000

We assumed constant scaling of expected revenue.
More realistic, we would have irregular increases stemming from dynamic pricing and customer behaviour.

Total revenue: $R(x) = R_0(x) + R_1(x) + R_2(x)$

$$\begin{aligned} \text{maximise} \quad & R(x) = R_0(x) + R_1(x) + R_2(x) \\ \text{subject to} \quad & x_0 + x_1 + x_2 \leq 7 \\ & x_0,\ x_1,\ x_2 \geq 0 \end{aligned}$$

Result obtained via backward recursion

$$(x_0, x_1, x_2) = (5, 0, 2)$$

with a total revenue $R(x) = 12\ 000$.

Consider the variation

$$\begin{aligned} \text{maximise} \quad & R(x) = R_0(x) + R_1(x) + R_2(x) \\ \text{subject to} \quad & x_0 + x_1 + x_2 \leq 7 \\ & x_0,\ x_1,\ x_2 \geq 0 \\ & x_1 \geq x_0 \end{aligned}$$

Result obtained via backward recursion

$$(x_0, x_1, x_2) = (2, 4, 1)$$

with a total revenue $R(x) = 10\ 000$.

This can be generalised:

$$\text{maximise} \qquad R(x) = \sum_{j=1}^{M} R_j(x)$$

$$\text{subject to} \qquad \sum_{j=1}^{M} x_j \leq n_{\text{max}}$$

$$x_j \geq 0 \qquad (j = 1, 2, \ldots, M)$$

(possibly other constraints)

Improvements that can be made on the model by considering more ticket classes

Let

- $x_0 =$ ticket class A (not vaccinated)
- $x_1 =$ ticket class B (not vaccinated)
- $x_2 =$ ticket class A (vaccinated)
- $x_3 =$ ticket class B (vaccinated)

# tickets x_j	$R_0(x)$	$R_1(x)$	$R_2(x)$	$R_3(x)$
0	0	0	0	0
1	1200	1500	960	1200
2	2400	3000	1920	2400
3	3600	4500	2880	3600
4	5000	5600	4000	4480
5	8000	7700	6400	6160
6	8500	8700	6800	6960
7	9200	10000	7360	8000

Our model can be used to determine approximately how many tickets should be given to vaccinated passengers such that maximized revenue is not compromised.

This is done by solving the optimisation problem

$$
\begin{aligned}
\text{maximise} \quad & R(x) = R_0(x) + R_1(x) + R_2(x) + R_3(x) \\
\text{subject to} \quad & x_0 + x_1 + x_2 + x_3 \leq 7 \\
& x_0,\ x_1,\ x_2 \geq 0 \\
& x_0 \leq x_2 \\
& x_1 \leq x_3
\end{aligned}
$$

Solving via Optimal Control yielded interpretable results almost immediately (in terms of the modelling process).

Linear Programming Approach

Revenue optimisation

Let

- $p_j \in \mathbb{R}_{\geq 0}$ be the price of an individual class of ticket, and
- $x_j \in \mathbb{N}$ be the number of that class of tickets sold,

with $j = 1, 2, \ldots, d$.

Construct the ticket price and ticket quantity vectors

$$p = \begin{bmatrix} p_1 \\ \vdots \\ p_d \end{bmatrix} \in \mathbb{R}^d_{\geq 0}, \qquad x = \begin{bmatrix} x_1 \\ \vdots \\ x_d \end{bmatrix} \in \mathbb{N}^d$$

The revenue function may then be defined as

$$f(x_1, \ldots, x_d) = p_1 x_1 + \cdots + p_d x_d = p^T x$$

Normalisation

Instead of working with the number of tickets per class, we can work with the ratio of tickets per class compared to the total number of tickets (in terms of the aeroplane carrying capacity).

Let $x_{\max}$ denote the carrying capacity of the aeroplane.

Define

$$\bar{x}_j = \frac{x_j}{x_{\max}}$$

then $\bar{x}_j \in [0, 1] \subset \mathbb{R}_{\geq 0}$ and $\bar{x} \in \mathbb{R}^d_{\geq 0}$.

Constraints

Positivity

$$p \geq 0, \qquad \bar{x} \geq 0$$

Capacity

$$\frac{x_{\max}}{2} \leq \sum_{j=1}^{d} x_j \leq x_{\max} \qquad \iff \qquad \frac{1}{2} \leq \sum_{j=1}^{d} \bar{x}_j \leq 1$$

Linear Programming Problem

$$\begin{aligned} \text{maximise} \quad & f(x) = p^T x \\ \text{subject to} \quad & x_j \geq 0 \\ & \sum_j x_j \leq x_{\max} \\ & \sum_j x_j \geq \frac{x_{\max}}{2} \end{aligned}$$

$$\begin{aligned} \text{maximise} \quad & f(\bar{x}) = p^T \bar{x} \\ \text{subject to} \quad & \bar{x}_j \geq 0 \\ & \sum_j \bar{x}_j \leq 1 \\ & \sum_j \bar{x}_j \geq \frac{1}{2} \end{aligned}$$

Basic ticket data for LP

Start with a base ticket price (BTP), based on a real example.

Considerations:

- One flight only
- BTP is for one adult
- Taxes and fees are given (not derived from base ticket price)
- “Everything” else derived from base ticket price.

Ticket pricing

Discounts:

- Child ($\sim$ 75% of BTP)
- Infant ($\sim$ 10% of BTP)
- Vaccinated ($\sim$ 20% of BTP)

Penalties:

- Flexible booking dates (30% of BTP)

Booking fee: Either discount or penalty?

- Online booking R27
- In-person booking R250 + VAT = R287.5

Ticket pricing (cont.)

Final Ticket Price (FTP):

$$\text{FTP} = \text{BTP} + \text{Taxes \& Fees} + \text{Penalties} - \text{Discounts}$$

Assign true/false values for the following questions:

- Is the booking online?
- Is the person a child?
- Is the person an infant?
- Is the person vaccinated?
- Are the booking dates flexible?

This generates $2^5 = 32$ possible different ticket “types” base on one BTP and one given “taxes and fees” value.

Discard infeasible combinations.

For example, we assumed that infants and children cannot be vaccinated.

FTP	Online?	Flexible?	Vaccinated?	Child?	Infant?
5795.5	0	0	0	0	0
5285.5	0	0	1	0	0
5540.5	0	0	0	0	1
3883.0	0	0	0	1	0
6560.5	0	1	0	0	0
6050.5	0	1	1	0	0
6305.5	0	1	0	0	1
4648.0	0	1	0	1	0
5535.0	1	0	0	0	0
5025.0	1	0	1	0	0
5280.0	1	0	0	0	1
3622.5	1	0	0	1	0
5790.0	1	1	1	0	0
6045.0	1	1	0	0	1
4387.5	1	1	0	1	0

Solving

$$\begin{aligned} \text{optimise} \quad & f(\bar{x}) = p^T \bar{x} \\ \text{subject to} \quad & \bar{x}_j \geq 0 \\ & \sum_j \bar{x}_j \leq 1 \\ & \sum_j \bar{x}_j \geq \frac{1}{2} \end{aligned}$$

with this data yields a trivial, yet expected result:

Sell all the seats at the maximum price (R6 560.50)
Revenue will be R642 929 on a 98-seater plane.

However, this is not "realistic":

- booked in-person
- flexible travel dates
- unvaccinated (that might be probable)

Extra constraints

We imposed extra constraints:

- Ratio of children and infants $\leq 10\%$
- Vaccinated passengers $\geq 50\%$
- Online bookings $\geq 75\%$

seat ratio	FTP	EI-90	EI-70
0.00	5795.50	0.00	0.00
0.00	5285.50	0.00	0.00
0.00	5540.50	0.00	0.00
0.00	3883.00	0.00	0.00
0.25	6560.50	24.50	18.25
0.00	6050.50	0.00	0.00
0.00	6305.50	0.00	0.00
0.00	4648.00	0.00	0.00
0.00	5535.00	0.00	0.00
0.00	5025.00	0.00	0.00
0.00	5280.00	0.00	0.00
0.00	3622.50	0.00	0.00
0.70	5790.00	68.60	51.10
0.05	6045.00	4.90	3.65
0.00	4387.50	0.00	0.00

These results tell us we need to sell

- 70% of tickets at R5 790 (booked online, flexible travel dates, vaccinated)
- 24% of tickets at R6 560.50 (booked in-person, flexible travel dates, unvaccinated)
- 1% of tickets at R6 045 (infant, booked in-person, flexible travel dates, unvaccinated)

to satisfy our extra constraints.

Revenue will be R599 277 on a 98-seater plane

Conclusions

Future work

- Better formulation of the ticket classes
- Incorporate dynamic pricing to revenue optimisation
- Ensure revenue optimisation models are working correctly

- Challenging problem with lots of dimensions
- Revenue could be optimised with various constraints taken into account
- Incentives (vaccination, discounts) could be used to increase revenues

Thank you for listening!

Questions?